CS5112: Algorithms and Data Structures for Applications

Lecture 13: Streaming algorithms

Ramin Zabih

Some figures from: Wikipedia/Google image search;
J. Leskovec, A. Rajaraman, J. Ullman: Mining of Massive Datasets, http://www.mmds.org

Administrivia

- Reminder: HW comments and minor corrections on Slack
  - Important announcements via email (best efforts)
- Q6 out tonight
- HW3 (and HW2!) delayed due to grading software issues
- **Anonymous** survey coming re: speed of course, etc.

Today

- Comments about the prelim!
- Six approximate algorithms for histograms and NN

Prelim comments

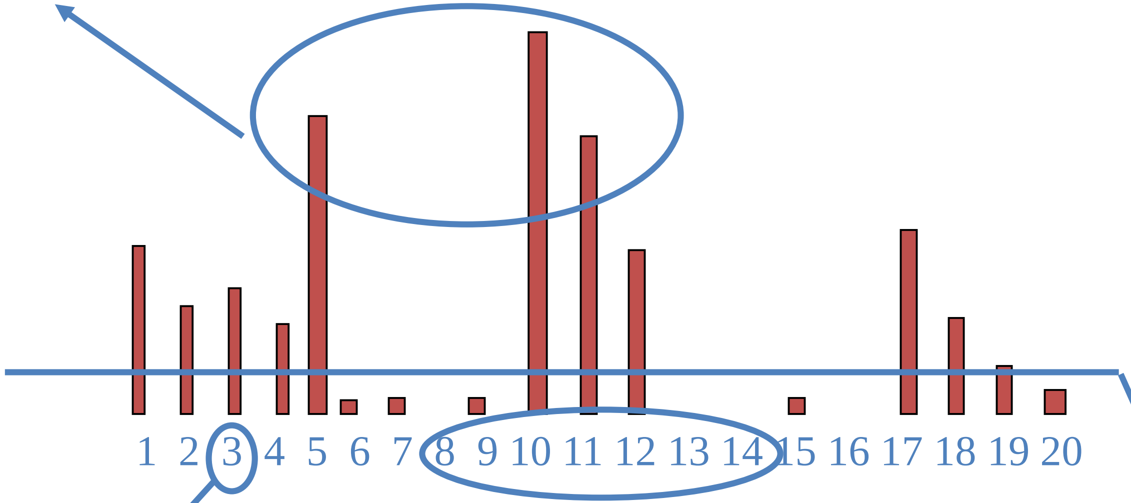
- NOTHING TODAY IS AUTHORITATIVE (YET)
- Closed book, multiple choice and short answer
- I will give example questions throughout today's lecture

Online histogram approximations

- Many AI/ML applications hinge on understanding the distribution of your input data
  - Classification is just one example
- Classically we assume that all the input data is available
  - You can run an offline algorithm over it
  - Then do NN classification, density estimation, etc.
- This assumption is often wrong: data comes streaming in
  - And you cannot afford to store the entire data set

Some natural histogram queries

Top-k most frequent elements



Find all elements with frequency > 0.1%

What is the frequency of element 3?

What is the total frequency of elements between 8 and 14?

How many elements have non-zero frequency?

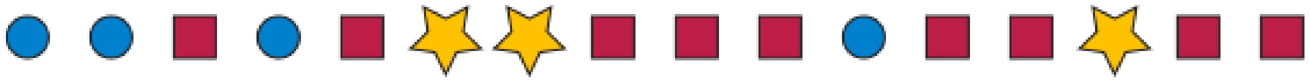
Why approximation?

- Suppose you want an (exact) histogram of your data
- This requires space linear in the number of data points
  - Which is intractable for many internet applications!
    - Examples: IP addresses for DDOS detection; most popular page/item to buy
- So instead we will use a small amount of space but solve the problem approximately
  - But, not precisely the same problem
- Instead of computing the histogram we will look at several key properties of a histogram we can efficiently approximate
  - Typically with constant or logarithmic space and time

Relevant quantities to compute

- **Majority**: if there is a single item comprising more than half the input stream, find it
- **Frequent items**: find all items that comprise more than a given percent ϕ of the input stream
  - Approximate version: find all items that comprise between $\phi - \epsilon$ and ϕ percent of the input stream (exact when $\epsilon = 0$)
  - Recent version: find and update the most recent popular items
- **Distinct items**: how many different items are there?

1. Boyer-Moore majority algorithm

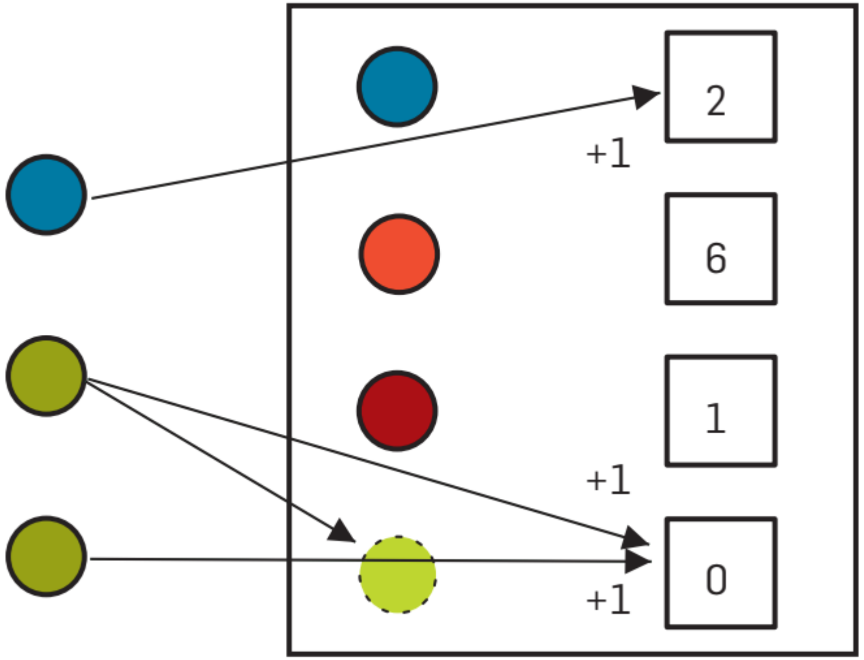


2. Misra-Gries

- Generalization of Boyer-Moore majority algorithm
- Store $k - 1$ counters, for a parameter k
  – Larger k means more space and accuracy
- Any item that appears more than $\frac{n}{k}$ times in the input stream of size n will be present when the algorithm terminates
- If $k = 1/\epsilon$ then each count is at most ϵn below its true value

Misra-Gries algorithm in action

Figure 2. Counter-based data structure: the blue (top) item is already stored, so its count is incremented when it is seen. The green (middle) item takes up an unused counter, then a second occurrence increments it.



Algorithm 1: FREQUENT(k)

$n \leftarrow 0$;
$T \leftarrow \emptyset$;
foreach i **do**
  $n \leftarrow n + 1$;
  if $i \in T$ **then**
    $c_i \leftarrow c_i + 1$;
  else if $|T| < k-1$ **then**
    $T \leftarrow T \cup \{i\}$;
    $c_i \leftarrow 1$;
  else forall $j \in T$ **do**
    $c_j \leftarrow c_j - 1$;
    if $c_j = 0$ **then** $T \leftarrow T \backslash \{j\}$;

Finding the Frequent Items in Streams of Data, Cormode and Hadjieleftheriou, CACM 52(10), October 2009

CORNELL TECH

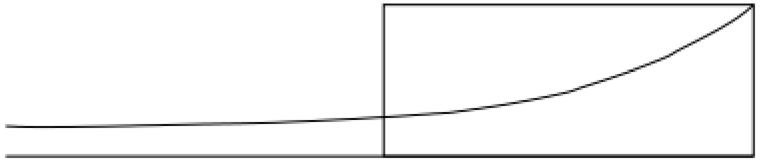
3. Find popular recent items

- Want to be able to naturally update this over time
  - Think of popular: movies, shopping items, web pages, etc.
- We could run, e.g., Misra-Gries on a sliding window
  - This is both impractical and wrong
- Wrong because the importance of an item should not “fall off a cliff” when it moves outside of our window

Weighted average in a sliding window

- Computing the average of the last k inputs can be viewed as a dot product with a constant vector $v = \left[\frac{1}{k}, \frac{1}{k}, \dots, \frac{1}{k}\right]$
- Sometimes called a box filter
  - Easy to visualize
- This is also a natural way to smooth, e.g., a histogram
  - To average together adjacent bins, $v = [\frac{1}{3}, \frac{1}{3}, \frac{1}{3}]$
- This kind of weighted average has a famous name

Decaying windows



- Let our input at time t be $\{a_1, a_2, \ldots, a_t\}$
- With a box filter over all of these elements we computed

$$\frac{1}{t}\sum_{i=0}^{t-1} a_{t-i}$$

- Instead let us pick a small constant c and compute

$$\hat{a}_t = \sum_{i=0}^{t-1} a_{t-i}(1-c)^i$$

Easy to update this

- Update rule is simple, let the current dot product be $\hat{a}_t$
$$\hat{a}_{t+1} = (1-c)\hat{a}_t + a_{t+1}$$
- This downscales the previous elements correctly, and the new element is scaled by $(1-c)^0 = 1$
- This avoids falling off the edge
- Gives us an easy way to find popular items

4. Popular items with decaying windows

- We keep a small number of weighted sum counters
- When a new item arrives for which we already have a counter, we update it using decaying windows, and update all counters
- How do we avoid getting an unbounded number of counters?
- We set a threshold, say ½, and if any counter goes below that value we throw it away
- The number of counters is bounded by $\frac{2}{c}$

How many distinct items are there?

- This tells you the size of the histogram, among other things
- To solve this problem exactly requires space that is linear in the size of the input stream
  - Impractical for many applications
- Instead we will compute an efficient estimate via hashing