CS5112: Algorithms and Data Structures for Applications

Lecture 3: Hashing

Ramin Zabih

Some figures from Wikipedia/Google image search

Cornell University

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Administrivia

- Web site is: https://github.com/cornelltech/CS5112-F18
  - As usual, this is pretty much all you need to know
- Quiz 1 out today, due Friday 11:59PM
  - Very high tech!
  - Coverage through Greg's lecture on Tuesday
- TA's and consultants coming shortly
- We have a slack channel

Today

- Clinic this evening (here), Greg on hashing
- Associative arrays
- Efficiency: Asymptotic analysis, effects of locality
- Hashing
- Additional requirements for cryptographic hashing
- Fun applications of hashing!
  - Lots of billion-dollar ideas

Associative arrays

- Fundamental data structure in CS
- Holds (key,value) pairs, a given key appears at most once
- API for associative arrays (very informal!)
  - Insert(k,v,A)->A’, where A’ has the new pair (k,v) along with A
  - Lookup(k,A)->v, where v is from the pair (k,v) in A
- Lots of details we will ignore today
  - Avoiding duplication, raising exceptions, etc.
- “Key” question: how to do this fast

How computer scientists think about efficiency

- Two views: asymptotic and ‘practical’
- Generally give the same result, but math vs engineering
- Asymptotic analysis, a.k.a. “big O”
  – Mathematical treatment of algorithms
  – Worst case performance
  – Consider the limit as input gets larger and larger

Big O notation: main ideas

- 2 big ideas:
  - **[#1]** Think about the worst case (don't assume luck)
  - **[#2]** Think about all hardware (don't assume Intel/AMD)
- Example: find duplicates in array of n numbers, dumbly
  - For each element, scan the rest of the array
  - We scan $n-1, n-2, \ldots$ elements **[#2]**
  - So we examine $\sum_{i=1}^{n-1} i$ elements, which is $\frac{(n-1)(n-2)}{2}$
  - Which is ugly… but what happens as $n \to \infty$? **[#1]**
- Write this as $O(n^2)$

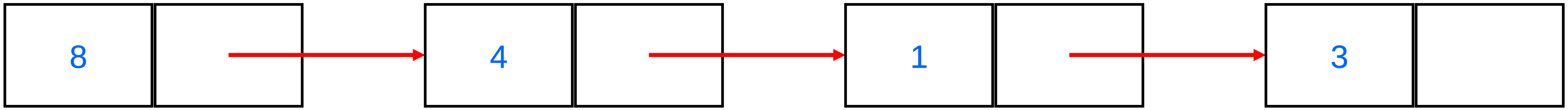
Some canonical complexity classes

- Constant time algorithms, $O(1)$
  - Running time doesn't depend on input
  - Example: find the first element in an array
- Linear time algorithms, $O(n)$
  - Constant work per input item, in the worst case
  - Example: find a particular item in the array
- Quadratic time algorithms, $O(n^2)$
  - Linear work per input item, such as find duplicates
- Clever variants of quadratic time algorithms, $O(n \log n)$
  - A few will be discussed in the clinic tonight

Big O notation and its limitations

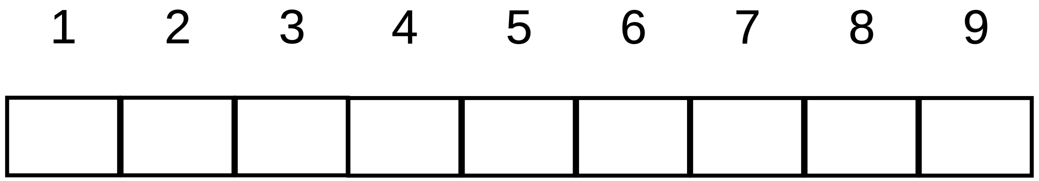
- Overall, practical performance correlates very strongly with asymptotic complexity (= big O)
  - The exceptions to this are actually famous
- Warning: this does not mean that on a specific input an $O(1)$ algorithm will be faster than an $O(n^2)$ one!

Linked lists



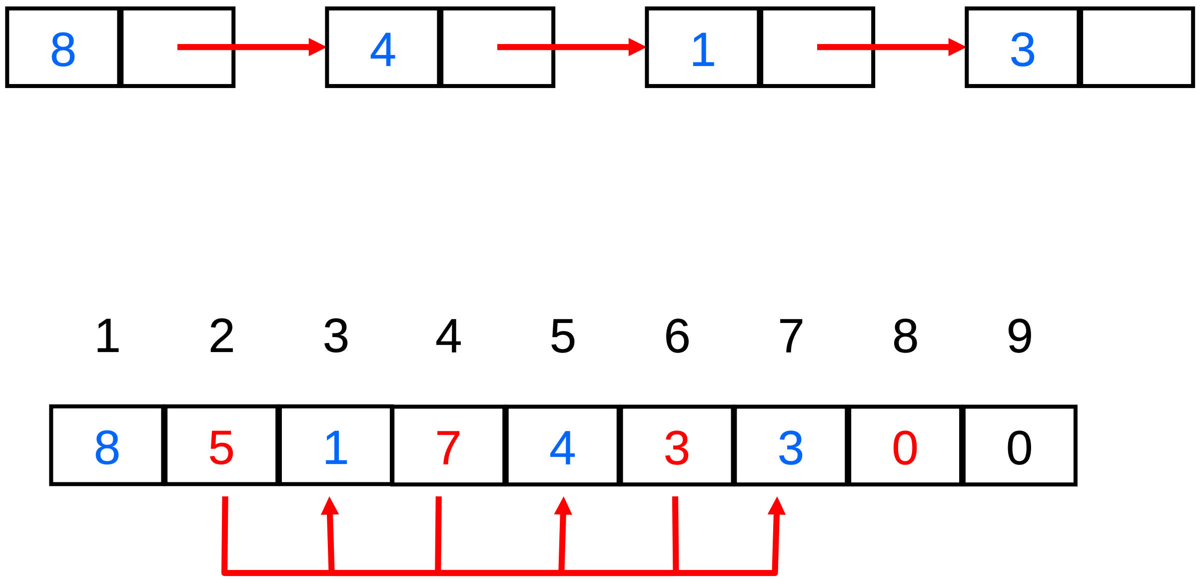
Linked lists as memory arrays

- We'll implement linked lists using a memory array
  - This is very close to what the hardware does



- A linked list contains "cells"
- A value, and where the next cell is
  - We will represent cells by a pair of adjacent array entries

Example



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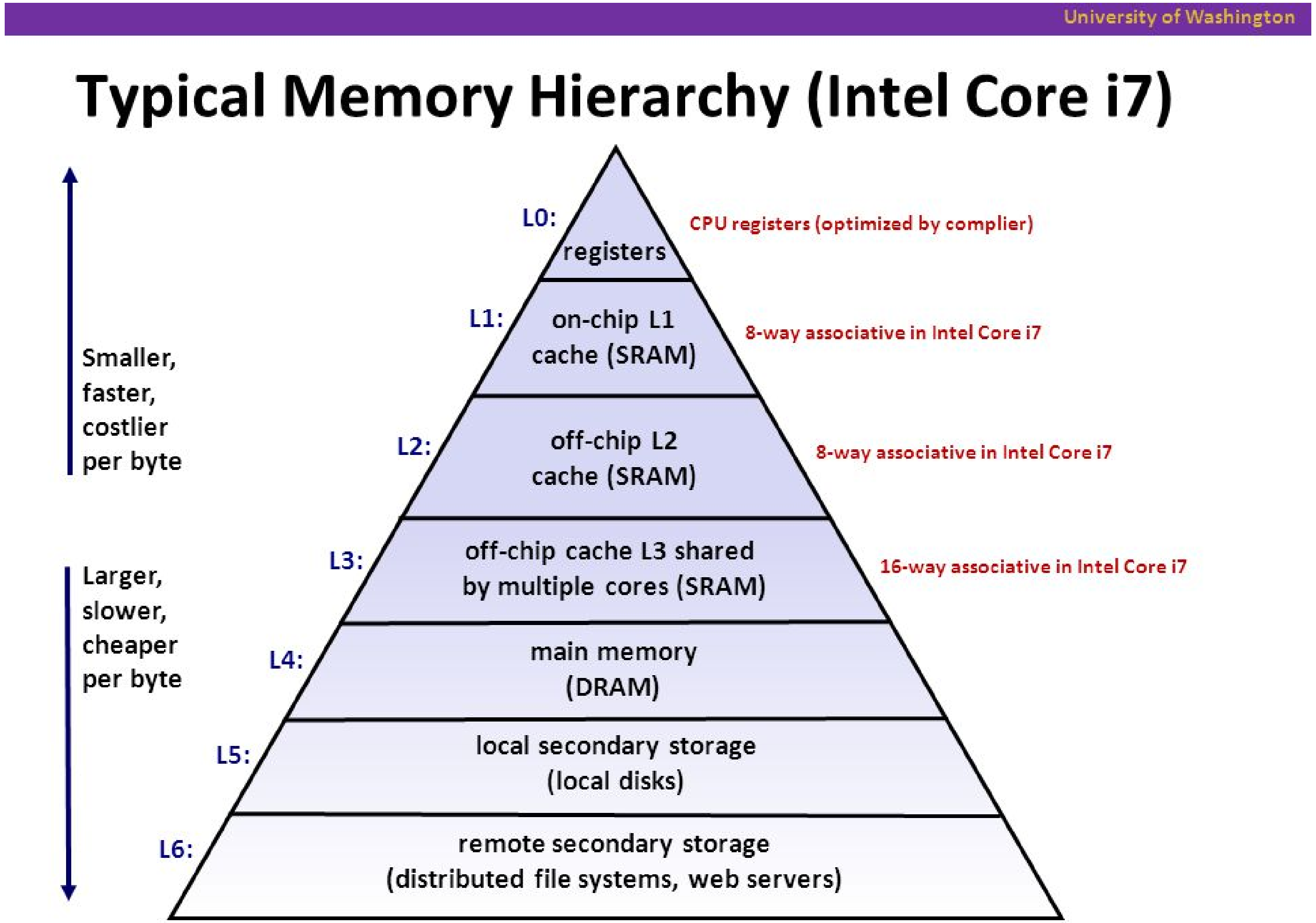
Locality and efficiency

- Locality is important for computation due to physics
  - The amount of information you can pack into a given area
- The hardware is faster when your memory references are local in terms of time and space
- Time locality: access the same data repeatedly, then move on
- Space locality: access nearby data (in memory)

Memory hierarchy in a modern CPU

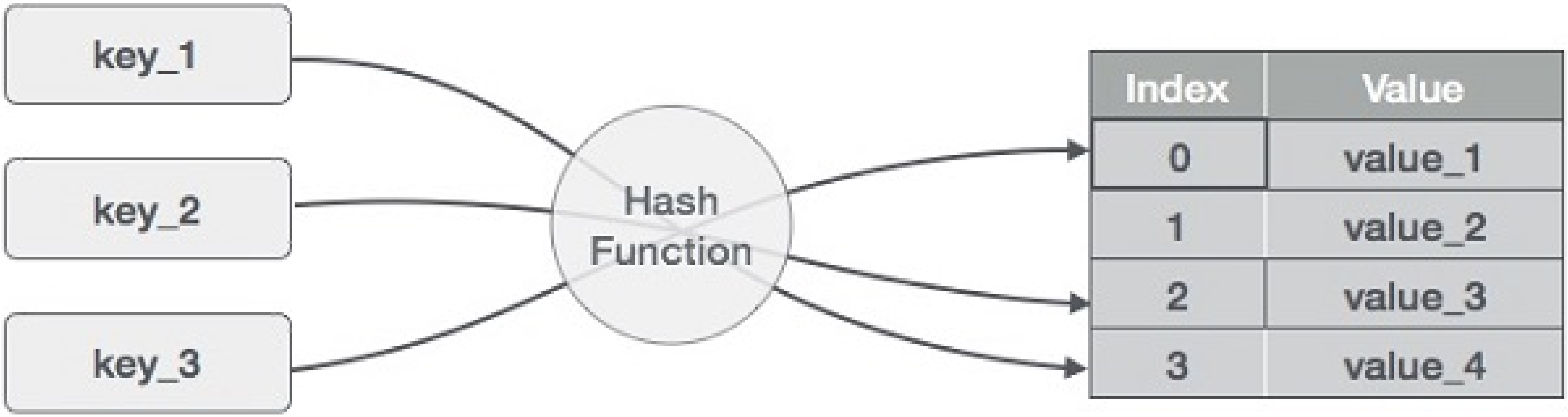


Complexity of associative array algorithms

DATA STRUCTURE	INSERT	LOOKUP
Linked list	$O(n)$	$O(n)$
Binary search tree (BST)	$O(n)$	$O(n)$
Balanced BST	$O(\log n)$	$O(\log n)$

So, why use anything other than a hash table?

Hashing in one diagram



What makes a good hash function?

- Almost nobody writes their own hash function
  - Like a random number generator, very easy to get this wrong!
- Deterministic
- Uniform
  - With respect to your input!
  - Technical term for this is entropy
- (Sometimes) invariant to certain changes
  - Example: punctuation, capitalization, spaces

Examples of good and bad hash functions

- Suppose we want to build a hash table for CS5112 students
- Is area code a good hash function?
- How about zip code?
- Social security numbers?
  - https://www.ssa.gov/history/ssn/geocard.html
- What is the best and worst hash function you can think of?

Cryptographic hashing

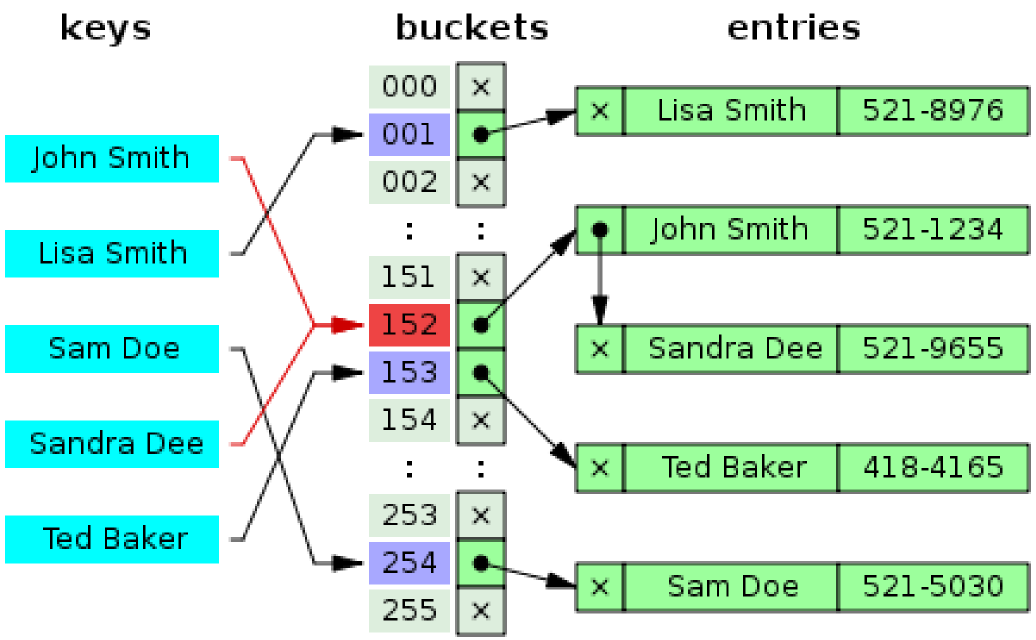
- Sample application: bragging that you've solved HW1
  - How to show this without sharing your solution?
- Given the output, **hard** to compute an input with that output
  - Given $m = hash(s)$ hard to find $s'\colon m = hash(s')$
  - Sometimes called a 1-way function
- Given the input, hard to find a matching input
  - Given s hard to find $s'\colon hash(s) = hash(s')$
- Hard to find two inputs with same output: $hash(s) = hash(s')$

What does "hard to find" mean?

- Major topic, center of computational complexity
- Loosely speaking, we can't absolutely prove this
- But we can show that if we could solve one problem, we could solve another problem that is widely believed to be hard
  - Because lots of people have tried to solve it and failed!
- This proves that one problem is at least as hard as another
  - "Problem reduction"

Handling collisions

- More common than you think!
  - Birthday paradox
  - Example: 1M buckets and 2,450 keys uniformly distributed
  - 95% chance of a collision
- Easiest solution is chaining
  - E.g. with linked lists



Now for the fun part…

What cool stuff can we do with hashing?

Rabin-Karp string search

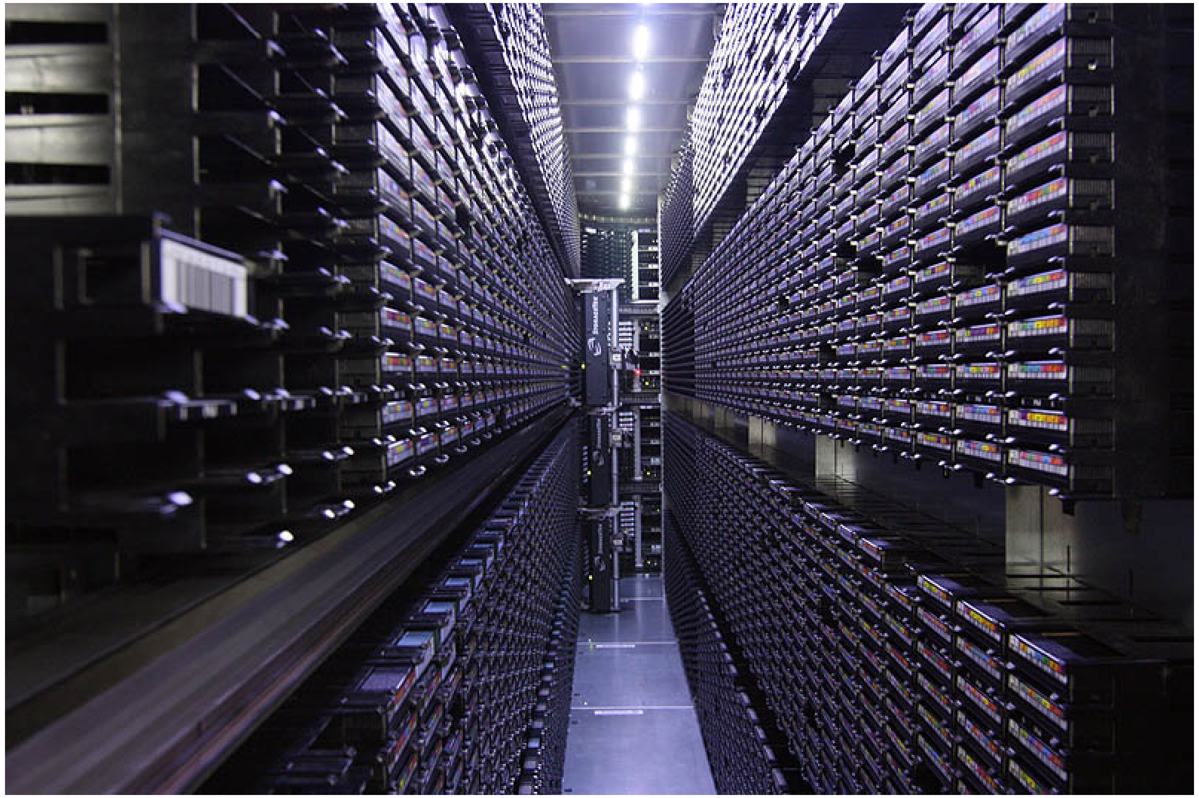
- Find one string (“pattern”) in another
  - Naively we repeatedly shift the pattern
  - Example: To find “greg” in “richardandgreg” we compare greg against “rich”, “icha”, “char”, etc. (‘shingles’ at the word level)
- Instead let’s use a hash function h
- We first compare h(“greg”) with h(“rich”), then h(“icha”), etc.
- Only if the hash values are equal do we look at the string
  - Because $x = y \Rightarrow h(x) = h(y)$ (but not $\Leftarrow$ of course!)

Rolling hash functions

- To make this computationally efficient we need a special kind of hash function h
- As we go through “richardandgreg” looking for “greg” we will be computing h on consecutive strings of the same length
- There are clever ways to do this, but to get the flavor of them here is a naïve way that mostly works
  – Take the ASCII values of all the characters and multiply them
  – Reduce this modulo something reasonable

Large backups

- How do we backup all the world's information?
- Tape robots!
- VERY SLOW access

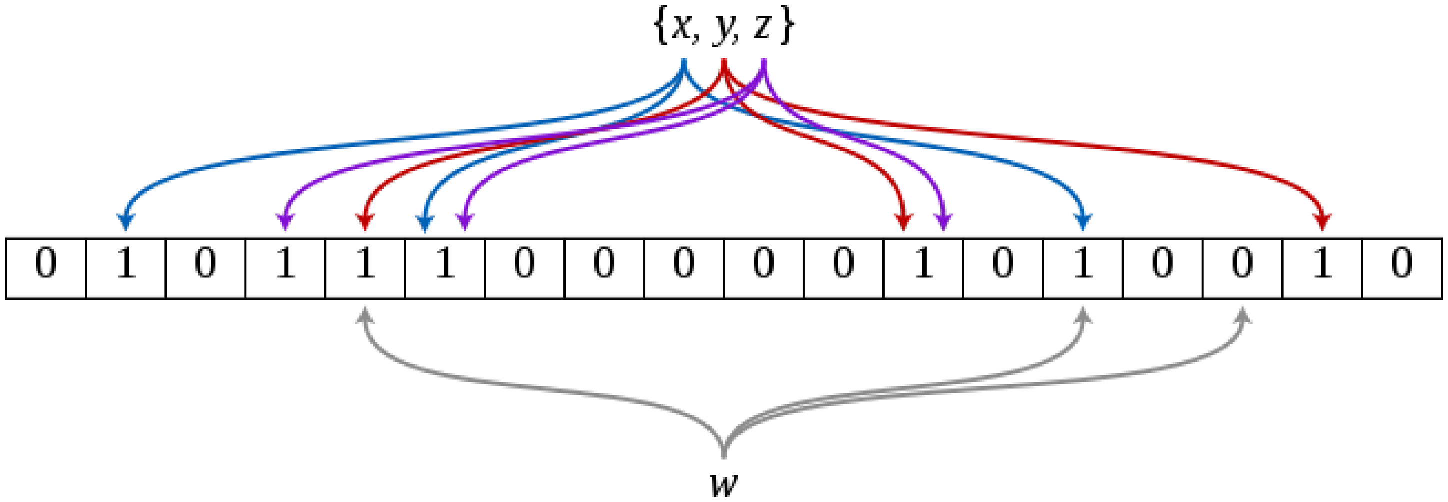


Bloom filters

- Suppose you are processing items, most of them are cheap but a few of them are very expensive.
  - Can we quickly figure out if an item **is** expensive?
  - Could store the expensive items in an associative array
  - Or use a binary valued hash table?
    - Efficient way to find out if an item **might be** expensive
- We will query set membership but allow *false positives*
  - I.e. the answer to $s \in S$ is either 'possibly' or 'definitely not'
- Use a few hash functions h_i and bit array A
  - To insert s we set $A[h_i(s)] = 1 \; \forall \, i$

Bloom filter example

- Example has 3 hash functions and 18 bit array
- $\{x, y, z\}$ are in the set, w is not

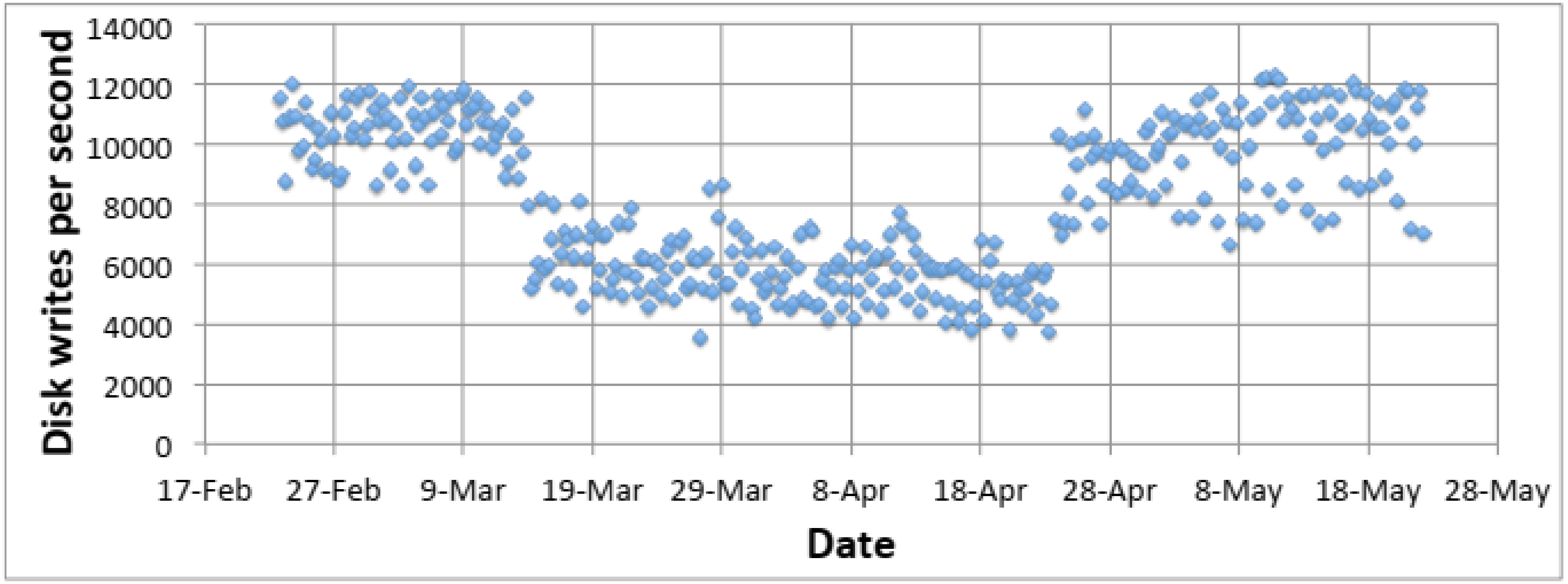


- Figure by David Eppstein, https://commons.wikimedia.org/w/index.php?curid=2609777

Application: web caching

- CDN's, like Akamai, make the web work (~70% of traffic)
- About 75% of URL's are 'one hit wonders'
  – Never looked at again by anyone
  – Let's not do the work to put these in the disk cache!
    - Cache on second hit
- Use a Bloom filter to record URL's that have been accessed
- A one hit wonder will not be in the Bloom filter
- See: Maggs, Bruce M.; Sitaraman, Ramesh K. (July 2015), "Algorithmic nuggets in content delivery" (PDF), *SIGCOMM Computer Communication Review*, New York, NY, USA,**45** (3): 52–66

Bloom filters really work!



- Figures from: Maggs, Bruce M.; Sitaraman, Ramesh K. (July 2015), "Algorithmic nuggets in content delivery" (PDF), *SIGCOMM Computer Communication Review*, New York, NY, USA,**45** (3): 52–66

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Cool facts about Bloom filters

- You don’t need to build different hash functions, you can use a single one and divide its output into fields (usually)
- Can calculate probability of false positives and keep it low
- Time to add an element to the filter, or check if an element is in the filter, is independent of the size of the element (!)
- You can estimate the size of the union of two sets from the bitwise OR of their Bloom filters

MinHash

- Suppose you want to figure out how similar two sets are
  - Jacard similarity measure is $J(A,B) = \frac{|A \cap B|}{|A \cup B|}$
  - This is 0 when disjoint and 1 when identical
- Define $h_{min}(S)$ to be the element of S with the smallest value of the hash function h, i.e. $h_{min}(S) = \arg\min_{s \in S} h(s)$
  - This uses hashing to compute a set's "signature"
- Probability that $h_{min}(A) = h_{min}(B)$ is $J(A,B)$
- Do this with a bunch of different hash functions

MinHash applications

- Plagiarism detection in articles
- Collaborative filtering!
  - Amazon, NetFlix, etc.

Distributed hash tables (DHT)

- BitTorrent, etc.
- Given a file name and its data, store/retrieve it in a network
- Compute the hash of the file name
- This maps to a particular processor, which holds the file